Modelling the Specific Grinding Energy and Ball Mill Scale-up

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Abstract: We propose a new model for the prediction of the specific grinding energy, which proved to approach very well the values calculated with the help of the Denver slide rule. The proposed model, combined with a model giving the ball-mill power draw, is used for the ball-mill scale up. Comparisons with the Bond procedure showed a good agreement for the prediction of the ball-mill dimensions, due to the fact that the proposed model is not sensitive to the various corrections associated with the Bond methodology.

Introduction

For the ball-mill scale up, the Bond work index $w_i$ together with the feed and product size (80% cumulative passing) were used for the estimation (from Bond’s law) of the specific grinding energy $w$. From the specific grinding energy $w$ (kWh/short ton) and the design tonnage rate (capacity) $T$ (short ton/h), the total mill power draw is calculated and then from manufacturers’ tables, charts or equations the number of mills and their dimensions (diameter, length) and are estimated for a particular power draw.

In addition to those previously mentioned, the Denver Equipment Company has proposed a circular rule (Denver slide rule), which is a particular nomograph for the selection of a ball-mill operating in wet closed grinding circuit. From this rule, for a given size reduction ratio $R$ (feed size $D_f$ and product size $d$), given ore hardness and given capacity $T$ (short ton/h), the mill power draw $P$ is calculated. The predicted power draw corresponds to a particular ball-mill size, performing a given ore size reduction.

Several researchers $^1$-$^8$ and mill manufacturers Nordberg $^9$, Morgardshammar $^{10}$ have worked with the power draw of tumbling mills and have proposed equations from which the mill power is

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determined, as a function of the mill operating conditions (fraction of mill filling \( f_L \) and fraction of the mill critical speed \( f_c \)), the apparent specific gravity of the charge \( \rho \) and its dimensions (diameter \( D \), length \( L \)).

In our previous work\(^{11}\), using dimensional analysis, we proposed an equation of the same form as those previously mentioned for the prediction of the mill power draw. The equation for the grinding mill power draw was also derived applying the torque theory, with respect to the mill rotational axis, for the charge centre of gravity\(^{11}\).

Recently, Morell\(^{12}\) made a significant contribution to the prediction of grinding mill power draw by proposing two different models (the C-Model and the simplified E-Model). The first model (theoretical) was based on the way the mill charge moves inside the mill and the second (empirical) is based on the first, but it contains fewer and simpler equations. The whole procedure leads to an extensive database of ball-, autogenous and semi-autogenous mills together with their associated power draws.

In the present paper an effort is made to propose a simple and efficient model for the calculation of the specific grinding energy and another one, in combination with the first model, for the determination of the mill power draw, in which the Bond work index \( w_i \) is also embodied. The proposed models are then used for ball-mill scale-up purposes.

**Model Development**

**Derivation of the equation for the calculation of the specific grinding energy \( w_m \) (soft ore)**

**Mathematical treatment**

The data used for the derivation of the new equation refer to medium (hardness) ore. They were obtained from Denver slide rule (Bulletin No. B2-B34).

Applying multiple linear regression analysis to 36 sets of data \((w_m, D_f \text{ and } d)\), the equation derived for medium ore gives the relationship between \( w_m \), \( D_f \) and \( d \).

The equation formed is of the general form:

\[
\begin{align*}
    w_m &= k_m f(D_f, d), \\
    f(D_f, d) &= D_f^{0.193} d^{-0.962} \\
    k_m &= 1.290 \text{ (constant, characteristic of the ore hardness)}. \\
\end{align*}
\]

Finally, Eq. (1) for the medium ore becomes:
\[ w_w = 1.290D_f^{0.193}d^{-0.962} \text{(kWh/short ton)} \] (2)

Afterwards, using data (24 sets) from Denver slide rule and keeping the same relationship (exponents) for \( D_f \) and \( d \), e.g. \( f(D_f, d) = D_f^{0.193}d^{-0.962} \), the predicted coefficients \( k \) (\( k_s \) and \( k_h \) for soft and hard ore, respectively) are:
\( k_s = 0.671 \) and \( k_h = 1.961 \).

Thus, for these cases the specific grinding energy \( w \) is given from:
\[ w_s = 0.671D_f^{0.193}d^{-0.962} \text{ (soft ore)} \] (3)

and
\[ w_h = 1.961D_f^{0.193}d^{-0.962} \text{ (hard ore)} \] (4)

In Fig. 1 a comparison is made between the specific grinding energy \( w \) values, which are determined from Denver slide rule, and those calculated from Eqs. (2), (3) and (4) for medium, soft and hard ores, respectively. From the distribution of points around the line of comparison \( y=x \), angle \( 45^\circ \), the good agreement of the results obtained from Eqs. (2), (3) and (4) and those obtained from the Denver rule is obvious.

![Graph comparing specific grinding energy](image)

**Fig.1.** Specific grinding energy \( w \) for medium, hard and soft ore predicted from the proposed model Eqs. (2), (3) and (4) versus \( w \) estimated from Denver slide rule.

Introduction of the Bond work index \( (w_i) \) as a parameter in the proposed equation.

It is known (personal communication with Denver Equipment Company) that there is a close relationship between the Bond work index \( (w_i) \) and the ore hardness designated by Denver. This relationship after Denver is given in Table 1.
Table 1. Relationship between the Bond work index \((w_i)\) and the ore hardness designated by Denver

<table>
<thead>
<tr>
<th>Denver ore-hardness</th>
<th>Bond work index ((w_i)), kWh/short ton</th>
<th>Coefficients (k)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Used for the calculation of (c) coefficient</td>
<td>Calculated from Eq. (5)</td>
</tr>
<tr>
<td>Soft</td>
<td>6.5</td>
<td>(0.671) 0.689 (2.68%)</td>
</tr>
<tr>
<td>Medium soft</td>
<td>9.0</td>
<td></td>
</tr>
<tr>
<td>Medium</td>
<td>12.0</td>
<td>(1.290) 1.272 (-1.40%)</td>
</tr>
<tr>
<td>Medium hard</td>
<td>15.0</td>
<td></td>
</tr>
<tr>
<td>Hard</td>
<td>18.0</td>
<td>(1.961) 1.908 (-2.70%)</td>
</tr>
</tbody>
</table>

Taking into account the data \((in italics)\) of Table 2, the constants of Eqs. (2), (3) and (4) can be efficiently approached from:

\[
k = c \ w_i = 0.106 \ w_i \quad (5)
\]

Thus, Eqs. (2), (3) and (4) are given now from the general expression:

\[
w = 0.106 \ w_i D_f^{0.193} d^{-0.962} \quad (kWh/short ton) \quad (6)
\]

or

\[
w = 0.1169 \ w_i D_f^{0.193} d^{-0.962} \quad (kWh/t) \quad (7)
\]

Introducing the reduction ratio \(R = (D_f / d)\) in Eqs. (6) and (7) and setting \(d\) in microns (\(\mu\)m), Eqs. (6) and (7) are transformed into:

\[
w = 21.5 \ w_i R^{0.193} d^{-0.769} \quad (kWh/short ton) \quad (8)
\]

or

\[
w = 23.70 \ w_i R^{0.193} d^{-0.769} \quad (kWh/t) \quad (9)
\]

Denver has reported that the slide rule must be applied for wet closed-grinding circuit calculations and predicts the ball-mill power draw for the reduction of a feed size \(D_f\) to product size \(d\). The power draw predicted corresponds to ball-mills of fixed dimensions. These mills can be selected with the help of the slide rule proposed.
Power Draw of Denver Ball Mills

In our previous work (Stamboltzis and Tsakalakis, 1993), we showed that the tumbling mills power draw $P$ is proportional to the product $D^{2.5} \times L$, where $D$ is the internal mill diameter and $L$ is the mill length. The values of this product are calculated changing the dimensions, given in the Denver slide rule, from feet (ft) to metres (m). The product $D^{2.5} \times L$ has dimensions $m^{3.5}$.

With application of linear least squares regression to 45 pairs $(P, D^{2.5} \times L)$ obtained from Denver slide rule, the linear equation, without the constant term ($y=bx$), is:

$$P = 12.767D^{2.5}L \text{ (HP)}$$

(10)

and if $P$ is given in kW, Eq. (10) becomes:

$$P = 9.524D^{2.5}L \text{ (kW)}$$

(11)

Equations (10) and (11) show a very good fit to the data. The correlation coefficient is $r = 0.9996$.

Eq. (11) gives the Denver ball-mill power draw in kW as a function of its dimensions.

In Fig. 2, a comparison is made between the ball-mill power draw values, which are determined from the Denver slide rule, and those calculated from Eq. (11). From the distribution of points around the *line of comparison* ($y=x$, angle 45°), the good agreement of the results obtained from Eq. (11) and those obtained from the Denver rule is obvious.

It is known (Arbiter and Harris, 1980; Rowland and Kjos, 1980; Arbiter and Harris, 1982; Harris and Arbiter, 1982; Dor and Bassarear, 1982; Turner, 1982; Austin et al., 1992; Stamboltzis and Tsakalakis, 1993) that the power drawn from tumbling mills (rod, ball-mills, etc.) is given from the general equation:

$$P = kp f_L (1 - f_L) f_c D^{2.5}L$$

(12)

where $\rho$ is the apparent specific gravity (short ton/m$^3$) of the mill charge (balls or rods), $f_L$ the fraction of mill filling and $f_c$ the fraction of mill critical speed.
The values used by Denver (Bulletin B2-B34 p.30, 31 and 51) are:

\[ \rho = 4.9455 \text{ short ton/m}^3, \quad f_L = 0.45 \text{ and } f_c = 0.75. \]

Thus, we have:

\[ \rho f_L (1 - f_L) f_c = 0.9180 \quad (13) \]

therefore, from Eq. (10),

\[ k = \frac{12.767}{0.9180} = 13.91 \]

Replacing the value of \( k \) in Eq. (12), it gives:

\[ P = 13.91 \rho f_L (1 - f_L) f_c D^{2.5} L \quad (\text{HP}) \quad (14) \]

or

\[ P = 10.38 \rho f_L (1 - f_L) f_c D^{2.5} L \quad (\text{kW}) \quad (15) \]

If we set \( \rho = 4.5 \text{ t/m}^3 \) in Eq. (13), then Eq. (15) yields:

\[ P = 11.402 \rho f_L (1 - f_L) f_c D^{2.5} L \quad (\text{kW}) \quad (16) \]
Prediction of the mill dimensions for a given Bond work index $w_i$, feed size $D_f$ (80% cumulative passing), product size (80% cumulative passing) $d$ and for capacity $T$

\[ P = w_i T, \text{ (kW)} \]  

(17)

where $w$ is the specific grinding energy (kWh/short ton) and $T$ (short ton/h) is the mill capacity.

Substituting for $w$ from Eq. (6) into Eq. (17) yields:

\[ P = 0.106 w_i D_f^{0.193} d^{-0.962} T \]  

(18)

From Eqs. (13), (15) and (17):

\[ T d D_f^{0.962} 0.193 \times D_f 962.0193.05.3 106.0)9180.038.101( = \]  

(19)

where: $D$, $L$ (m), $w_i$ (kWh/short ton), $D_f$ and $d$ (mm) and $T$ (short ton/h).

The ball-mill dimensions (internal mill diameter $D$ and length $L$) are calculated from Eq. (19) for a given ($L/D$) ratio, for feed size $D_f$ and product size $d$ (mm), for known Bond work index $w_i$ (kWh/short ton) and for desirable capacity $T$ (short ton/h),

**Application for medium (hardness) ore**

($w_i = 12$ kWh/short ton) and for $D_f = 50.8$ mm, $d = 0.417$ mm and $T = 50$ short ton/h.

From the above data and for capacity $T = 50 \times 24 = 1200$ short ton/24h, using the Denver slide rule, the suitable ball-mill for this size reduction has dimensions $D \times L = 8' \times 12'$ or diameter $D = 2.44$ m and length $L = 3.66$ m.

Thus, the ratio $L/D = 12/8 = 1.5$.

From Eq. (19), replacing the above mentioned data and assuming that the ratio $L/D = 1.5$, results:

\[ D^{3.5} \cdot (1.5) = 11.13 \cdot 10^{-2} \cdot 12 \cdot 50.8^{0.193} \cdot 0.417^{-0.962} \cdot 50 = 33.06 \]

or, $D^{3.5} = 22.04$ and solving for $D$:

$D = 2.42$ m and $L = 1.5 \times 2.42 = 3.63$ m.

The calculated dimensions ($D \times L = 2.42$ m x 3.63 m) show a very good agreement with those ($D \times L = 2.44$ m x 3.66 m) predicted from the Denver slide rule.
Comparison between the Bond method (classic) and the Denver method

The comparison between the Bond and Denver methods is made with the help of data presented from Rowland (1982).

1st problem: (Rod and Ball-mill circuit)

Data: Rowland, 1982, pp. 427-429

Initial feed size 25 mm = 25000 μm (80% passing, 18 mm), product (80% passing, 0.175 mm).

Bond work index:
1. \( w_i \) in the 10 mesh (rod mill Bond work index): 13.2 kWh/short ton
2. \( w_i \) for 65 mesh (0.212 mm) product: 11.7 kWh/short ton
3. \( w_i \) for 100 mesh (0.147 mm) product: 12.1 kWh/short ton
4. \( w_i \) for 325 mesh (0.045 mm) product: 14.0 kWh/short ton
5. Throughput \( T = 500 \) t/h (metric tonnes/h)

Procedure: Rod mill grinding from 18 mm to 1.2 mm and then ball-mill grinding of the rod mill product from 1.2 mm to 0.175 mm.

Determination of the ball-mill dimensions

Feed size (80% passing): \( D_f = 1.2 \) mm = 1200 μm
Product size (80% passing): \( d = 0.175 \) mm = 175 μm (≅ 80 mesh)

The Bond work index for product \( d = 0.175 \) mm is about \( w_i \approx 12 \) kWh/short ton, since \((0.212 \text{ mm} + 0.147 \text{ mm})/2 = 0.1795 \text{ mm} = 0.175 \text{ mm}\).

From Eq. (6) and for \( w_i = 12 \) kWh/short ton, the specific grinding energy is \( w = 7.05 \) kWh/short ton or \( w = 7.77 \) kWh/t.

For capacity \( T = 500 \) t/h, the required mill power draw must be 7.77 kWh/t×500 t/h of = 3885 kW (equation 17). Therefore, if two (2) ball-mills are used, each one must draw 3885/2 = 1942.5 kW

From Eq. (11), the internal mill diameter \( D \) (m) for each mill is calculated as:

\[
D = \left[1942.5/(1.5 \times 9.524)\right]^{1/3.5} = 4.07 \text{ m}, \text{ (for a ratio } L/D = 1.5\).
\]

Thus, its length should be: \( L = 1.5 \times 4.07 = 6.1 \text{ m}\)

Thus, for the above defined size reduction, two ball-mills \((D \times L = 4.07 \text{ m} \times 6.1 \text{ m})\) are required, which will operate under the conditions proposed by Denver \((\rho = 4.5 \text{ t/m}^3, f_L = 0.45 \text{ and } f_c = 0.75 \text{ or 75%})\)
But, for the same size reduction and taking into account the various corrections, the Bond method (Rowland, 1982) gives two (2) ball-mills \((D \times L = 3.93 \text{ m} \times 5.79 \text{ m})\), with operational conditions \(\rho = 4.65 \text{ t/m}^3\), \(f_L = 0.4\) and \(f_c = 0.689\) or 68.9%.

For that reason, a correction coefficient for the factor \(\rho f_L (1-f_L) f_c\) must be applied. The value of the correction coefficient is:

\[
\frac{[4.65 \times 0.4 \times (1-0.4) \times 0.689]}{[4.5 \times 0.45 \times (1-0.45) \times 0.75]} = 0.9205
\]

Thus, in this case the ball-mill dimensions predicted from Eq. (11) change to:

Diameter \(D = \left(\frac{1942.5 \times 0.9205}{1.5 \times 9.524}\right)^{1/3.5} = 3.97 \text{ m}\) (for the same ratio \(L / D = 1.5\)),

and the length is: \(L = 1.5 \times 3.97 = 5.96 \text{ m}\).

These dimensions \((D = 3.97 \text{ m}, L = 5.96 \text{ m})\) approach very well those of the Bond method \((D = 3.93 \text{ m}, L = 5.79 \text{ m})\).

2nd problem: (Simple ball-mill circuit).

Data: (Rowland, 1982, p. 430)

1. Ball-mill feed preparation in closed circuit (feed size 100% cumulative passing 12 mm, and 80% cumulative passing 9.4 mm).
2. Product size –0.175 mm (80% cumulative passing).
3. Assuming that the Bond work index from –9.4 mm to –0.175 mm is (roughly) \(w = \frac{(13.2+12)}{2} = 12.6 \text{ kWh/short ton}\)
4. Throughput 500 t/h.

For the above data Eq. (6) gives: \(w = 11.01 \text{ kWh/short ton} \) or \(w = 12.13 \text{ kWh/t}\)

For a capacity 500 t/h, the total required mill power is 12.13 kWh/t \(\times\) 500 t/h = 6050 kW.

Thus, if two (2) ball-mills are used, the power drawn from each Denver mill must be \((6050/2) = 3025 \text{ kW}\).

The internal mill diameter \(D \text{ (m)}\) calculated from Eq. (11) is:

\[
D = \left[\frac{3025}{(1.25 \times 9.524)}\right]^{(1/3.5)} = 4.87 \text{ m} \text{ (for a ratio } L / D = 1.25)\).
\]

The mill length should be: \(L = 1.25 \times 4.87 = 6.09 \text{ m}\).

Thus, for the above defined size reduction two ball-mills \((D \times L = 4.87 \text{ m} \times 6.09 \text{ m})\) are required, which will operate under the conditions adopted from Denver \((\rho = 4.5 \text{ t/m}^3, f_L = 0.45 \text{ and } f_c = 0.75 \text{ or } 75\%)\).

But, for the same size reduction and taking into account the various corrections, the Bond method gives two (2) ball-mills \((D \times L = 4.85 \text{ m} \times 6.1 \text{ m})\) and operational conditions \(\rho = 4.65 \text{ t/m}^3, f_L = 0.4\) and \(f_c = 0.689\) or 68.9%.

For that reason, a correction coefficient for the factor \(\rho f_L (1-f_L) f_c\) must be applied.
The value of the correction coefficient is:
\[
\frac{[4.65 \times 0.4 \times (1-0.4) \times 0.689]/[4.5 \times 0.45 \times (1-0.45) \times 0.75]}{= 0.9205}.
\]

Thus, the ball-mill dimensions from Eq.(11) change to the following:

\[
Diameter \ D = [3025 \times 0.9205]/(1.25 \times 9.524)]^{(1/3.5)} = 4.75 \ m \ \text{(for a ratio } L/D = 1.25)\]

The mill length must be: \( L = 1.25 \times 4.75 = 5.94 \ m \).

These dimensions \( (D = 4.75 \ m, L = 5.94 \ m) \) approach fairly close those of the Bond method \( (D = 4.85 \ m, L = 6.1 \ m) \).

3rd problem: (Regrinding circuit using Ball-mill).

Data: (Rowland, 1982, p. 433)

1. Ball-mill feed size -210 μm (-70 mesh).
2. Product size -45 μm (-325 mesh).
3. Bond work index for a -45 μm (-325 mesh) product: \( w_i = 14.0 \ kWh/\text{short ton} \)
4. Throughput 40 t/h.

For the above data, Eq. (6) gives:

\[
w = 21.70 \ kWh/\text{short ton} \ \text{or } w = 23.91 \ kWh/t
\]

For the 40 t/h, the required mill power is 23.91 kWh/t \( \times 40 \ t/h = 956.4 \ kW \).

For a ratio \( L/D = 1.92 \), the internal mill diameter \( D \) (m) from Eq. (11) is:

\[
D = [956.4/(1.92 \times 9.524)]^{(1/3.5)} = 3.10 \ m,
\]

and the mill length is: \( L = 1.92 \times 3.10 = 5.95 \ m \)

Thus, for the above defined size reduction, one ball-mill \( (D \times L = 3.10 \ m \times 5.95 \ m) \) is required, which will operate under the conditions adopted from Denver \( (\rho = 4.5 \ t/m^3, f_L = 0.45 \text{ and } f_c = 0.75 \text{ or } 75\% ) \).

But, for the same size reduction and taking into account the various corrections, the Bond (classic) method gives one ball-mill \( (D \times L = 3.05 \ m \times 5.83 \ m) \) and operational conditions \( \rho = 4.65 \ t/m^3, f_L = 0.4 \text{ and } f_c = 0.75 \text{ or } 75\% \).

For that reason, a correction coefficient for the factor \( \rho f_L(1-f_L)/f_c \) must be applied.

The value of the correction coefficient is: \[
\frac{[4.65 \times 0.4 \times (1-0.4) \times 0.75]/[4.5 \times 0.45 \times (1-0.45) \times 0.75]}{= 1.002}
\]

The value of the correction shows that there is no need for new mill calculations.
4th problem: (Ball-mill grinding circuit)

Data: (Rowland, 1982, p. 434)

1. Feed material (iron ore) for the production 400 t/h of fine material for pellets.
2. Ball-mill feed size -400 μm (80% cumulative passing).
3. Product size of -26 μm (80% cumulative passing).
4. Bond work index: \( w_i = 26.0 \) kWh/short ton

From Eq.(6), the specific grinding energy is:

\[ w = 77.34 \text{ kWh/short ton or } w = 85.23 \text{ kWh/t} \]

For throughput 400 t/h production, the total required ball-mill power is: 85.23 kWh/t \( \times \) 400 t/h \( = \) 34092 kW. If six (6) ball-mills are used, the power drawn from each mill is: \( (34092/6) = 5682 \) kW.

For a ratio \( L/D = 1.54 \), the internal mill diameter \( D (m) \) is calculated as:

\[ D = \left[ \frac{5682}{(1.54 \times 9.524)} \right]^{1/3.5} = 5.49 \text{ m}. \]

The mill length is: \( L = 1.54 \times 5.49 = 8.45 \text{ m}. \)

Thus, for the above defined size reduction, six (6) ball-mills \( (D \times L = 5.49 \text{ m} \times 8.45 \text{ m}) \) are required, which will operate under the conditions adopted from Denver \( (\rho = 4.5 \text{ t/m}^3, f_i = 0.45 \text{ and } f_c = 0.75 \text{ or } 75\%). \)

But, for the same size reduction and taking into account the various corrections, the Bond (classic) method gives six (6) ball-mills \( (5.49 \text{ m} \times 8.45 \text{ m}) \) and operational conditions \( \rho = 4.65 \text{ t/m}^3, f_i = 0.4 \text{ and } f_c = 0.675 \text{ or } 67.5\%). \)

For that reason, a correction coefficient for the factor \( \rho f_L (1-f_L) f_c \) must be applied.

The value of the correction coefficient is: \( [4.65 \times 0.4 \times (1-0.4) \times 0.675]/[4.5 \times 0.45 \times (1-0.45) \times 0.75] = 0.9018. \)

Thus, the ball-mill dimensions given from Eq. (11) change to:

\[ Diameter \ D = \left[ \frac{5682 \times 0.9018}{(1.54 \times 9.524)} \right]^{1/3.5} = 5.33 \text{ m (for a ratio } L / D = 1.54). \]

The mill length is: \( L = 1.54 \times 5.33 = 8.21 \text{ m}. \)

These dimensions \( (D = 5.33 \text{ m}, L = 8.21 \text{ m}) \) are fairly close to those predicted from the Bond method \( (D = 5.49 \text{ m}, L = 8.45 \text{ m}) \).

The dimensions estimated refer to ball-mills operating under wet grinding conditions. These ball-mills are of overflow type (Overflow discharge). If these ball-mills were of grate type (wet grate discharge), the power for the calculation of their dimensions must be increased by about 16% (Rowland, 1982).
Conclusions

In the present work, equations were derived, which give:

- the specific grinding energy $w$, as a function of: the feed $D_f$ (mm) and product size $d$ (mm) (both refer to 80% cumulative passing), the Bond work index $w_i$ (kWh/short ton) or, as a function of the size reduction ratio $R = D_f / d$, of the Bond work index $w_i$ and the product size $d$,
- the ball-mill power draw as a function of its dimensions: internal mill diameter $D$ and length $L$,
- the ball-mill power draw as a function of the feed $D_f$ (mm) and the product size $d$ (mm), the Bond work index $w_i$ (kWh/short ton) and the mill throughput $T$ (short ton/h),
- the ball-mill dimensions ($D$ and $L$), when not only $D_f$, $d$, $w_i$, and $T$, but also the mill operating conditions ($\rho$, $f_o$, and $f_c$) are known and assuming the value of the ($L/D$) ratio.

From this work it was shown that these equations approach very well the values calculated with the help of the Denver slide rule. They represent the mathematical expression of Denver slide rule, which is not always available.

It was additionally shown, that the ball-mill dimensions predicted from the above methodology are almost equal to those of the Bond method. This fact is very important, because the various corrections, associated with the Bond methodology, are not necessary and the model developed can be used as an alternative method for ball scale-up purposes.

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